Playing around with piano music

R.M. Hartkamp

May 14, 2011

Some people have an absolute hearing and can identify individual notes just by listening to a piece of music. For those who cannot do that, all is not lost. Fourier transforming a signal allows one to identify the frequencies that occur in a signal. Unfortunately, it becomes messy really fast. Transforming the whole piece at once would give a collection of frequencies without any information about the order or the rhythm. A smarter approach is needed if one would like to analyze a longer piece of music. This should not be too difficult though, since the key strokes are clear pulses in the signal. The signal gives us the rhythm and transforming a short period of time after each pulse gives us the notes. However, only a brief impression is given here, I first analyze the first key strokes of a piece of piano music and later show the amplitude spectrum of a longer piece of music.

Figure 1 shows the signal of a key stroke which damps out in time.

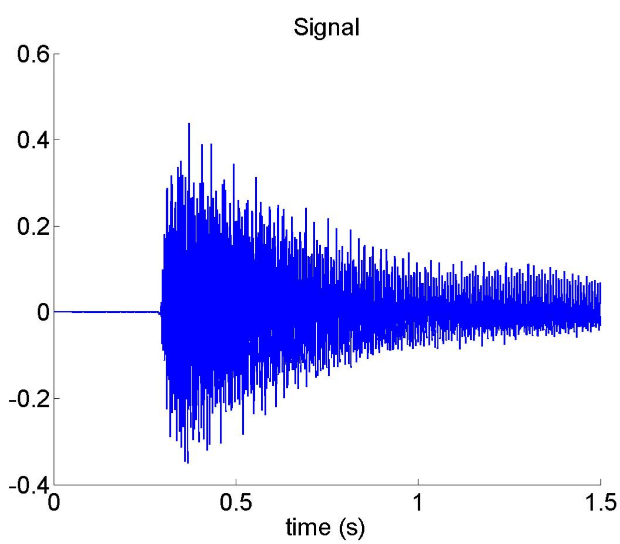

Figure 1: Signal

This short sound signal hides a lot of information, as the Fourier transform shows in Figure 2. The first five peaks are found at the frequencies: (1) 131.2, (2) 195.8, (3) 262.4, (4) 311.9 and (5) 393.7 Hz. The peaks with a frequency higher than 393.7 Hz are higher

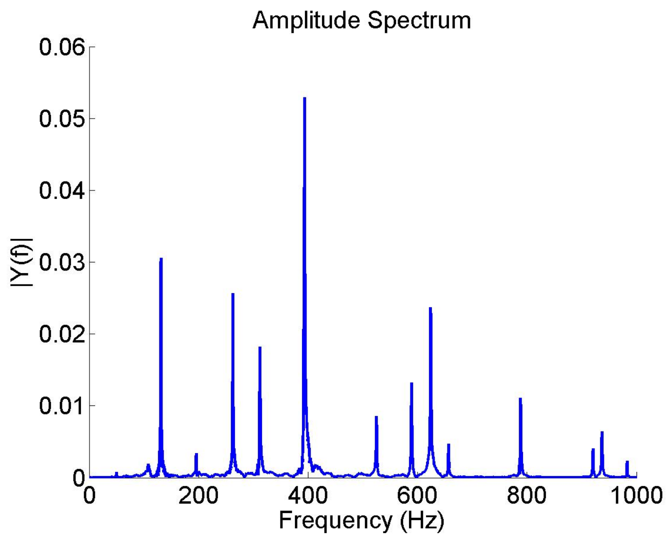

Figure 2: Fourier spectrum

harmonics (I use some prior knowledge here). The higher harmonics can just as well be added to the sheet music, it just requires large hands and enough fingers. This possibility is disregarded here.

Each note on the piano has a different frequency. These frequencies are given by:

$$F(n) = 440 \times 2^{\frac{n-49}{12}} \tag{1}$$

If we invert Eq. (1), we can calculate which notes correspond to the frequencies we found in our signal.

$$n = 12 \log_2(\frac{F}{440}) + 49 \tag{2}$$

Substituting the frequencies that we found in our spectrum into equation Eq. (2) gives key numbers 28, 35, 40, 43 and 47 as a result. If we count from left to right on the keyboard, we find exactly what is also on the sheet music (Figure 3).

Figure 3: Original sheet music of the signal.

A preview of a dynamic analysis is shown in Figure 4. A Fourier transformation is applied at fixed time intervals, this would not be the most elegant way of doing it, but it is effective. Furthermore, one could make an effort to reduce the noise in the signal by using wavelet transforms. The transformed signal in Figure 4 is only meant to demonstrate the idea, hence the short signal. Finally, the ratio between the frequencies of different keys on the piano is

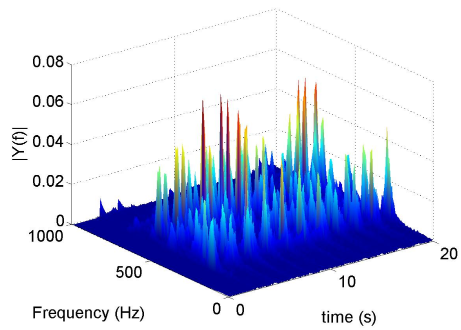

Figure 4: First 20 seconds of a prelude by Bach.

given by $2^{n/12}$, with n the difference in the number of the keys. Knowing this, it would not be terribly difficult to translate a piece of music to another key by multiplying the frequency domain by certain given factor.